STATISTICAL ANALYSIS OF THE NATURE OF COLOMBIAN GROSS NATIONAL PRODUCT COMPONENTS

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ABSTRACT

The analysis of a trend's nature is fundamental in Economics. This paper reviews such nature on a macroeconomic aggregate series of the Colombian quarterly GNP over a period of time between the 1977 First Quarter, and the 1997 Fourth Quarter.

To this end, several methodologies including the pioneer papers of Nelson & Plosser, Watson, Campbell & Mankiw y Cochrane[i] are used. This survey complements previous work conducted in Colombia for the same series over a different period including the paper by Alejandro Gaviria and José Darío Uribe[ii]. Finally, permanence before recessionary and expansionary shocks is analyzed based on Beaudry & Koop's[iii] article.

KEY WORDS: Trend- stationary, difference-stationary, permanence, impulse response functions

INTRODUCTION, MATERIALS AND METHODS

The research papers previously mentioned studied the behavior of American macroeconomic series components. For the specific case of the American Gross

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National Product (GNP), the results on the nature of its non-stationarity were contradictory. Later in Beaudry & Koop's paper, permanent expansions and transitory recessions were found.

DEVELOPMENT AND RESULTS

The National Planning Department provided the Colombian quarterly GNP series. Its historical period began the 1977 First Quarter, and ended the 1997 Fourth Quarter. The available percentages of subsequent dates are used for the calculation of nonlinear impulse responses.

The natural logarithmic series used is then seasonally adjusted to substract the deterministic natural station.

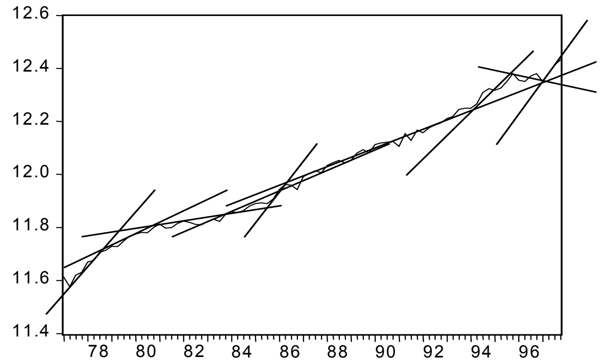

Figure 1
QUARTERLY GNP (1977-1Q:1997-4Q)

Review of the series did not show only one slope. On the contrary, it showed 9 periods of "growth" that are different regarding both inclination and duration. 19 recessions were found, where common duration was one quarter once a year in most cases. The result was:

Initial year and quarter of the recession	Recession duration in quarters	% of quarterly GNP change during the recession	Initial Year and Quarter of Recession	Recession Duration in quarters	% of Quarterly GNP change during the recession
1977:2	1	-0,30797975			
1979:2	1	-0,00579662	1988:4	1	-0,06819149
1980:3	1	-0,00466773	1989:4	1	-0,09408139
1981:2	1	-0,13575526	1991:1	1	-0,14612782
1982:2	3	-0,12125947	1991:3	1	-0,20936645
1983:3	1	-0,07377217	1992:1	1	-0,08085715
1984:2	1	-0,01500791	1994:1	1	-0,00473402
1985:3	1	-0,02471691	1995:1	1	-0,05249208
1986:3	2	-0,15655522	1996:1	2	-0,243555
1987:4	1	-0,06391499	1997:1	1	-0,23105898

Table 1: Recessions during the period

I. Approximation of Nelson and Plosser: In a preliminary analysis, the detrending series was modeled through ARMA (Auto Regressive Moving Average) models, while the correlogram of the first difference suggested that the natural logarithm for the quarterly GNP could be an integrated logarithm of order 1. Due to previous results, it was necessary to use the Dickey-Fuller test. The result of the definite estimation was:

$$\Delta y_t = -0.0056517 - 0.001562 y_{t-1} + \sum_{i=1}^{4} \Delta y_{t-i} + \varepsilon_t \qquad (1)$$

Under the null hypothesis of γ=0, the t-statistical associated to γ was 0.203061, while the critical value for the Dickey-Fuller test to a significance of 5% was -2.8981. Therefore, the GNP could be modeled by a random walk with drift.

The previous analysis determines that the natural logarithm for the quarterly GNP is DS in the first difference.

II. Permanence Analysis:

a) Non-Parametric Permanence Analysis

Cochrane proposes a permanence measure that measures the size of a random walk process. The measure is determined by the following equation:

$$V_k = (\sigma_1^2)^{-1}(Var(y_t - y_{t-k}))/ k) = (\sigma_k/\sigma_1)^2 \qquad (2)$$

Where σ_1^2 is the variance for the first process difference, and σ_k^2 is 1/k times the variance of its k differences.

Pure processes will be analyzed (random walk with drift and the stationary process around the trend) to start the study of this measure. If the process is a random walk with drift, the Cochrane's measure will be constant throughout its time horizon, as shown below:

$$V_k= (\sigma_\varepsilon/\sigma_1)^2=1 \qquad (3)$$

While in a stationary process around the trend (TS), the Cochrane's measure is:

$$V_k= 2(k)^{-1}(\sigma_\varepsilon/\sigma_1)^2=1/k \qquad (4)$$

This will decrease quickly to zero. The persistence measure will find a value range between zero and infinite. It is evident that this measure has little power for a stationary process around the trend since the long-term measure space is constrained to a sole value (zero). Thus, the results near zero should be cautiously taken (it is important to say that a DS process, with a small walk random size is better represented by a TS process).

Based on the fact that a process that has a unit root can be decomposed into two components: a permanent component and a transitory one, that is to say, a random walk and a stationary component, Cochrane used the results of Beveridge and Nelson's (B&N) [iv] decomposition, and those of more generalized decompositions (where the correlation between the components, is not restricted to be exact), used by Watson to find his permanence measure. The permanent component variance (z_t), is given by:

$$\Delta y_t = \Delta z_t + \Delta c_t$$
$$\mathrm{Var}(\Delta z_t) = \sigma^2_{\Delta z t} = |A(1)|^2 \sigma^2_\varepsilon \tag{5}$$

The permanent component of y_t is called the stochastic trend by B&N, and it represents the random walk process. The previous result is more important in Cochrane's article because it finds a relationship between the random walk variance and gain. Cochrane also determined that the random walk variance is related to the k difference variance. The relationship is shown as follows:

$$\lim_{k \to \infty} \sigma^2_k = \sigma^2_{\Delta z t} \tag{6}$$

As we can see, they are the same on the infinite horizon. This result finds the magnitude of the random walk of any process, which is equal to the variance of k difference on an infinite horizon. Under this horizon, Cochrane's measure is:

$$V_k = \frac{(A(1))^2 \sigma^2_\varepsilon}{\sigma^2_1} \tag{7}$$

which is the same as the weighted gain on the variance of the first process difference. The last result was used by Campbell & Mankiw to find a permanence measure based on gain. Therefore, they defined the following:

$$A(1) = \sqrt{\frac{V_k}{1 - R^2}} \tag{8}$$

which is the variance fraction, and is predictable from the process' historical behavior. Then:

$$R^2 = 1 - \frac{\sigma^2_\varepsilon}{\sigma^2_1} \tag{9}$$

This measure is always greater than or equal to Cochrane's measure, which is a lower limit of gain. Campbell and Mankiw use the sample estimator ρ of R, which is the coefficient of order 1 auto-correlation and is given by:

$$\hat{\rho}_j = \left(\frac{T}{T-j} \right) \left(\frac{\sum_{t=j+1}^{T} \left(\Delta y_t \Delta y_{t-j} \right)}{\sum_{t=j+1}^{T} \left(\Delta y_t \right)^2} \right) \tag{10}$$

Cochrane's result obtained on the "infinite" horizon will be found empirically on a k moment where its estimator finds stability. It will degenerate in a moment after k, where the variance of k differences, does not represent the real value of the parameter, because the remaining sample will be decreasing as k increases, and sample variance will tend to zero. The results of the two measures are shown on the table below:

Cochrane's measure											
Period (1977 : 1Q - 1997 : 4Q)											
Quarter	1	2	3	4	5	6	7	8	9	10	11
V(k)	1	0,6981	0,5542	0,3749	0,4320	0,4902	0,4454	0,4004	0,4003	0,4391	0,4227
Non-parametric Gain provided by Campbell & Mankiw											
Period (1977:1Q - 1998:4Q)											
Quarter	1	2	3	4	5	6	7	8	9	10	11
A(k)	1	0,8370	0,7457	0,6133	0,6584	0,7014	0,6685	0,6339	0,6338	0,6638	0,6513

Table 2: Cochrane's measure y and non-parametric gain

b) Linear parametric permanence analysis: ARIMA (Auto Regressive Integrated Moving Average) models were used to represent the first difference of the natural logarithm of the quarterly GNP seasonally adjusted. The methodology of Campbell & Mankiw proposes that the p and q order, of the $\phi(L)$ and $\theta(L)$ polynomials respectively, can not be above 3.

A wrong statistical specification results when a TS process is differentiated, because a unit root is generated in the MA polynomial (a process that cannot be reversed). The **robustness of** the methodology allows us to infer the existence or non-existence of this wrong specification. The following development is shown in order to prove the previous statement:

$$\phi(L)y_t = \mu + bt + \varepsilon_t \tag{11}$$

takinga differenceon eachside

$$\begin{aligned} \phi(L)(1-L)y_t &= (1-L)(\mu + bt + \varepsilon_t) \\ \phi(L)(1-L)y_t &= \mu - \mu + bt - b(t-1) + \varepsilon_t - \varepsilon_{t-1} \\ \phi(L)\Delta y_t &= b + \varepsilon_t - \varepsilon_{t\text{-}1} = b + (1-L)\varepsilon_t \end{aligned} \tag{12}$$

In the following table, the polynomial roots do not display wrong specifications:

Parametrization	AR Roots	MA Roots
0,1		-.05
0,2		-.02 -.31i -.02+.31i
0,3		.51+.71i .51 -.71i -.68
1,0	.06	
1,1	-.76	-.79
1,2	-.65	-.40 -.26i -.40+.26i
1,3	-.56	.17 -.45 -.44i .45+.44i
2,0	.47 -.37	.
2,1	1.00 -.22	.97
2,2	1.00 -.70	.98 -.59
2,3	.96 -.76	.72 -.26i .72+.26i -.87
3,0	.62 -.27 -.26i -27+.26i	
3,1	1.00 -.14+.39i -.14 .39i	.97
3,2	1.00 -.30 -.89	.97 -.99
3,3	.93 .46 -.67	.88 -.32i .88+.32i -.70

Table 3: Polynomial roots associated with ARIMA models estimated to the process according to Campbell and Mankiw's methodology

The impulse response functions allow us to measure innovation permanence. The definition arises intuitively as the difference between two different expressions of y_{t+n} that are the same until a time t. One of the expressions serves as reference, while the other one is affected by history (the historical shocks are innovations of the independent series of the process' general representation) and is hit by a shock. The impulse response definition is shown as follows:

$$I_Y(k, \delta, \omega_{t-1,\ldots,}\omega_{t-m}) = E[y_{t+n\,/}\,\varepsilon_t = \delta,.., \varepsilon_{t+n} = 0, \omega_{t-1,\ldots,}\omega_{t-m}] - E[y_{t+n} / \varepsilon_t = 0,.., \varepsilon_{t+n} = 0] \tag{13}$$

Where k represents the prediction period, δ represents the innovation, and ω_{t-i} represents the historical shocks. A general representation of the y_t process is expressed through the following equation:

$$y_t = \phi(L)y_t + \Omega_1(L)z_{1t} + \dots\dots\dots + \Omega_f(L)z_{f\,t} + \theta_q\varepsilon_t \tag{14}$$

Where ϕ, Ω_i and θ correspond to the polynomials governing the auto regressive part, to the independent variables, and to the residues respectively. (This representation can include **feedback** or not, if it includes it then the model is multivariate; if not, it can be either multiple or univariate). If the general representation does not contain independent variables, the y_t process can be approximated through ARIMA models. The impulse response function for the stationary ARMA process is:

$$I_Y(k,\delta) = \left(\sum_{j=0}^{k-1} \pi_j\right)\delta \tag{15}$$

If the size of the shock increases or decreases by any multiple or sub-multiple, the impulse response function will experience the same change (linearity property). If the shock polarity changes, the impulse response function will be exactly opposite to the original impulse response (symmetry property). Since ARIMA models are univariate (therefore they do not contain Ω_I polynomials), they are not affected in any way by (ω_{t-i}) historical innovations. This property is called independence in history. This type of impulse response function is called linear. A measure of permanence called gain can be found thusly:

$$A(1) = \frac{\theta(1)}{\varphi(1)} \tag{16}$$

The impulse response of different models are shown below, when the series is affected by a positive unit innovation. The reference periods are taken according to the original study. The permanences together with the gain observed below show permanences more like a random walk than a stationary trend process:

Order (p,q)	1	2	4	8	16	20	40	80	A(1)
0,1	1	1,04745	1,04745	1,04745	1,04745	1,04745	1,04745	1,04745	1,04745
0,2	1	1,045763	1,14247	1,14247	1,14247	1,14247	1,14247	1,14247	1,14247
0,3	1	0,648094	1,238893	1,238893	1,238893	1,238893	1,238893	1,238893	1,238893
1,0	1	1,059029	1,0625134	1,0627191	1,0627313	1,0627320	1,0627320	1,0627320	1,0627320
1,1	1	1,027798	1,0227734	1,0181429	1,0160396	1,0158589	1,0157665	1,0157661	1,0157661
1,2	1	1,15174	1,19733	1,22500	1,23092	1,23108	1,23112	1,23112	1,2311206
1,3	1	1,1615180	1,1592322	1,2074396	1,2127826	1,2128318	1,2128373	1,2128373	1,2128373
2,0	1	1,1040210	1,3285267	1,3859708	1,3888335	1,3888403	1,3888407	1,3888407	1,3888407
2,1	1	0,806858	0,8826508	0,9631295	1,1187267	1,1956416	1,5715747	2,2819405	11,284972
2,2	1	0,9114370	0,9646831	1,0422202	1,1675490	1,2268365	1,5099053	2,0183880	5,3068705
2,3	1	0,6133500	0,8265084	1,2273663	1,8753158	2,1246665	2,8823046	3,3153344	3,4011041
3,0	1	1,0678590	1,3812721	1,5147074	1,5380278	1,5384459	1,5385160	1,5385160	1,5385160
3,1	1	0,751824	0,7674159	0,8307999	0,9758154	1,0479668	1,4019777	2,0773917	12,458829
3,2	1	0,833064	0,9370546	1,0157954	1,1799187	1,2671110	1,7145952	2,5912835	20,211622
3,3	1	0,667503	0,6622434	1,2128354	2,0692945	2,3447998	2,9690574	3,1479640	3,1581896

Table 4: Impulse responses for k quarters after innovation

c) Non-linear Parametric Permanence Analysis:

The general representation of a process can be simulated by a transference function model. The bivariate case has the following representation:

$$y_t = \mu + \phi(L)\, y_{t-1} + \Omega(L)\, z_{t-1} + \theta(L)\varepsilon_t \qquad (17)$$

where y_t is the endogenous variable, and z_t is the exogenous variable (there is causality from z_t to y_t, but not vice versa, because it would represent a" feedback" between variables which is not allowed in these models). (In cases where there is feedback, VARMA (Vector Auto Regressive Moving Average) models are used, which are multivariate extensions of ARMA models).

The Ω (L) polynomial is called the transference function, since this polynomial allows us to model the causality relationship of the independent variable on the dependent one. As in ARIMA models, the ϕ(L) polynomial determines the stationarity of the process. It is also necessary for the series to be stationary in order to be treated. This methodology recommends differentiation in case of non-stationarity of the mean, for any series.

If the effects of the z_t shocks over y_t want to be known, the answer impulse function is used. It is defined by the $\Omega(L)/(1-\phi(L))$ and $\theta(L)/(1-\phi(L))$ infinite order polynomials. The non-linear impulse response function with m historical periods is:

$$I(k,\delta,\omega_{t-1},\ldots,\omega_{t-m}) = \left(\sum_{i=0}^{n-1}\pi_i\right)\omega_{t-1} + \ldots + \left(\sum_{i=0}^{n+m-2}\pi_i\right)\omega_{t-m} + \left(\sum_{i=0}^{n-1}\psi_i\right)\delta \qquad (18)$$

This impulse response function does not have the properties of the linear case allowing possible asymmetries, which are the product of the possible combinations among size, polarity of the shocks, and history.

Using the procedure proposed by Beaudry and Koop, depth, used as explanatory variable of the transference functions, is defined as:

$$Depth_t = \max\{Y_{t-j}\}_{j\geq 0} - Y_t \qquad (19)$$

Depth behaves as white noise. This allows us to model the series of the Colombian quarterly GNP natural logarithm, with a transference function, since feedback does not exist between the independent series and the explanatory series. The transference function model used is:

$$\phi(L)\Delta y_t = \mu + \Omega(L)Depth_{t-1} + \theta(L)\varepsilon_t \qquad (20)$$

Here, $\phi(L)$ polynomial is of p order, the θ (L) polynomial is of q order, and the Ω (L) polynomial of r order. Following the methodology used by Beaudry and Koop (which is based in the parametrization of Cambell and Mankiw), the parametrization models (p, q, r) were estimated, where the order of each polynomial cannot be higher than three and its sum cannot exceed 6. The number of models obtained according to this methodology is 56. Given the number of estimated models, only the best impulse response functions are analyzed based on their order. The good behavior of the selected models is emphasized; coefficients are statistically significant to any confidence level, and the residues are white noise except for the transference function (1,0,0).

Since recession duration is between 1 and 3 quarters, 3 scenarios are proposed with the same number of historical shocks. The historical shocks of interest are those related to the series recessions. The first historical shock is calculated averaging the first depth values corresponding to the first quarters of the recessions. The second historical shock is calculated averaging the second depth values that correspond to the second quarters of the recessions; and the third shock is calculated in a similar way.

Transfer functions with positive (expansionary) and negative (recessionary) unit shocks are estimated to analyze their differences. The permanence of recessionary and expansionary shocks, measured by non-linear impulse response functions based on the best models according to the range, are shown as follows:

Model	0,0,1		0,1,1		0,2,1		0,2,2		2,1,2		0,3,3	
Quarter	-	+	-	+	-	+	-	+	-	+	-	+
1	- 0,7359	0,7359	- 0,9636	1,0364	- 0,9636	1,0364	-0,9674	1,0326	-0,9817	1,0183	-0,9693	1,0307
2	- 0,7872	0,7872	- 0,9737	1,0465	- 0,9507	1,0235	-0,9709	1,0337	-0,4703	0,4955	-1,0915	1,1533
4	- 0,7872	0,7872	- 0,9737	1,0465	- 0,8801	0,9529	-0,8865	0,9494	-0,3870	0,4149	-0,7478	0,8114
8	- 0,7910	0,7910	- 0,9737	1,0465	- 0,8801	0,9529	-0,8865	0,9494	-0,3987	0,4262	-0,7478	0,8114
16	- 0,7911	0,7911	- 0,9737	1,0465	- 0,8801	0,9529	-0,8865	0,9494	-0,3990	0,4265	-0,7478	0,8114
20	- 0,7911	0,7911	- 0,9737	1,0465	- 0,8801	0,9529	-0,8865	0,9494	-0,3990	0,4265	-0,7478	0,8114
40	- 0,7911	0,7911	- 0,9737	1,0465	- 0,8801	0,9529	-0,8865	0,9494	-0,3990	0,4265	-0,7478	0,8114
80	- 0,7911	0,7911	- 0,9737	1,0465	- 0,8801	0,9529	-0,8865	0,9494	-0,3990	0,4265	-0,7478	0,8114
History is limited to first, second and third average historical period of depth into recession. History is the same for each model.												

Table 5: Response impulses estimated based on the best transference functions selected by Akike's criterion in parametrization order

MODEL (0,2,1)						
	1 period		1st and 2nd periods		1st 2nd and 3rd periods	
Quarter	-	+	-	+	-	+
1	-0,9718	1,0282	-0,9436	1,0564	-0,9153	1,0847
2	-0,9589	1,0153	-0,9306	1,0435	-0,9024	1,0718
4	-0,8883	0,9447	-0,8601	0,9730	-0,8318	1,0012
8	-0,8883	0,9447	-0,8601	0,9730	-0,8318	1,0012
16	-0,8883	0,9447	-0,8601	0,9730	-0,8318	1,0012
20	-0,8883	0,9447	-0,8601	0,9730	-0,8318	1,0012
40	-0,8883	0,9447	-0,8601	0,9730	-0,8318	1,0012
80	-0,8883	0,9447	-0,8601	0,9730	-0,8318	1,0012
These response impulses were found by applying a historical shock of 0,03577% which corresponded to the largest recession during the period 1977:1Q - 1997:4Q of the series (-0,30797975% quarterly, in the natural logarithm of the quarterly GNP). Shocks of this value depth are applied in the 1st, 2nd and 3rd periods.						

Table 6: Response impulses estimated based on the best transference functions selected by Akike's criterion

CONCLUSIONS

It was found out that the series resembles more a process of difference- stationary (DS) than a process of trend-stationary (TS). The results also showed a long-term permanence as compared to recessionary shocks. The behavior of the Colombian quarterly GNP seems to be better explained by theories associated to multiple balance.

There is asymmetry in permanence. The positive shocks (expansionary) show a highly permanent behavior, even above 100% of the initial shock. The permanence with negative shocks (recessionary) shows a permanent behavior below 100%. Asymmetry is accentuated when more and larger historical shocks hit the system. In the most extreme simulated case, we find differences of 25% in absolute value in the two types of shocks.

- The expansionary non-linear impulse response functions showed a permanence behavior similar to that found in the previous analyses the linear permanence. When there is permanence up to 40%, the response impulse result of model 2,2,1 agrees with Cochrane's measure, which estimates an approximate permanence between 40% and 104% of the shock. (The rest of transference functions, the best ARIMA models, and the non-parametric gain measure coincide in having a permanence measure between 65% and 104%).

- The recessionary shocks are permanent, and they are located between 39% and 98% of the innovation.

REFERENCES

[i] Nelson, C.R. and C.I. Plosser, 1981, Trends and random walks in macroeconomics time series: some evidence and implications, Journal of Monetary Economics 10,139-162.

Watson, Mark W. (1986). Univariate Detrending Methods with Stochastic Trends, Journal of Monetary Economics , XVIII. 49-75.

Campbell. J. H. and N. G. Mankiw. 1987. Are Output Fluctuations Transitory?. Quarterly Journal of Economics 102. 857-880.

Cochrane, J.B.. 1988. How big is a random walk in GNP? Journal of Political Economy 96. 893-920.

[ii] Gaviria A. y Uribe J.D. 1994. Choques exógenos y cambios estructurales. Colombia: 1936-1991, Universitas Económica, Vol IX No 1 Febrero 1994.

[iii]Beaudry, P. Koop, G. "Do recession permanently change output". Journal of Monetary Economics 31, 1998.

[iv] Beveridge, Stephen, and Nelson , Charles R. A New Approach to Decomposition of Economic Time Series into permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle. Journal of Monetary Economics. 7 (March 1981): 151-174.